FIR and IIR Filters

(1)

Digital Filters

FIR

Finite Impulse Response

→ Impulse response is of finite duration

i.e finite no. of non-zero terms

e.g $h(n) = \begin{cases} 2, & |n| \leq 4 \\ 0, & \text{elsewhere} \end{cases}$

→ Implemented using structures with no feedback i.e. non-recursive structures of all zeros

→ Present response depends on present and past input samples.

IIR

Infinite Impulse Response

→ it has infinite no. of non-zero terms.

e.g

$h(n) = a^n u(n)$

is non-zero for $n \geq 0$

→ Implemented using structures having feedback

i.e. recursive structures of poles & zero

→ Present response is a function of present and past N values of excitation and past values of response

Advantages of FIR over IIR.

→ exact linear phase
→ stable
→ design methods are linear
→ Realized efficiently in hardware
→ Filter start-up transients have finite duration.

FIR Filter Design using Window Techniques

The frequency response of any digital filter is periodic in nature & is given by

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-j\omega n} \quad \text{— (1)}$$

(Fourier series)

where

$$h(n) = \frac{1}{2\pi} \int_{0}^{2\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega \quad \text{— (2)}$$

The Fourier coeff. of $h(n)$ are identical to impulse response of a digital filter.

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2 main problems in implementing eq (1) are

→ Impulse response is of infinite dur'n

→ filter is non-causal and unrealisable.

∴ filter resulting from fourier series is unrealisable IIR filter.

Solution is

to convert infinite dur'n impulse response into finite dur'n impulse response by truncating infinite series at $n = \pm N$.

But this will result in undesirable oscillations in passband & stopband of the filter.

These oscillations are removed / reduced by using time-limited weighting function $w(n)$, called as window function.

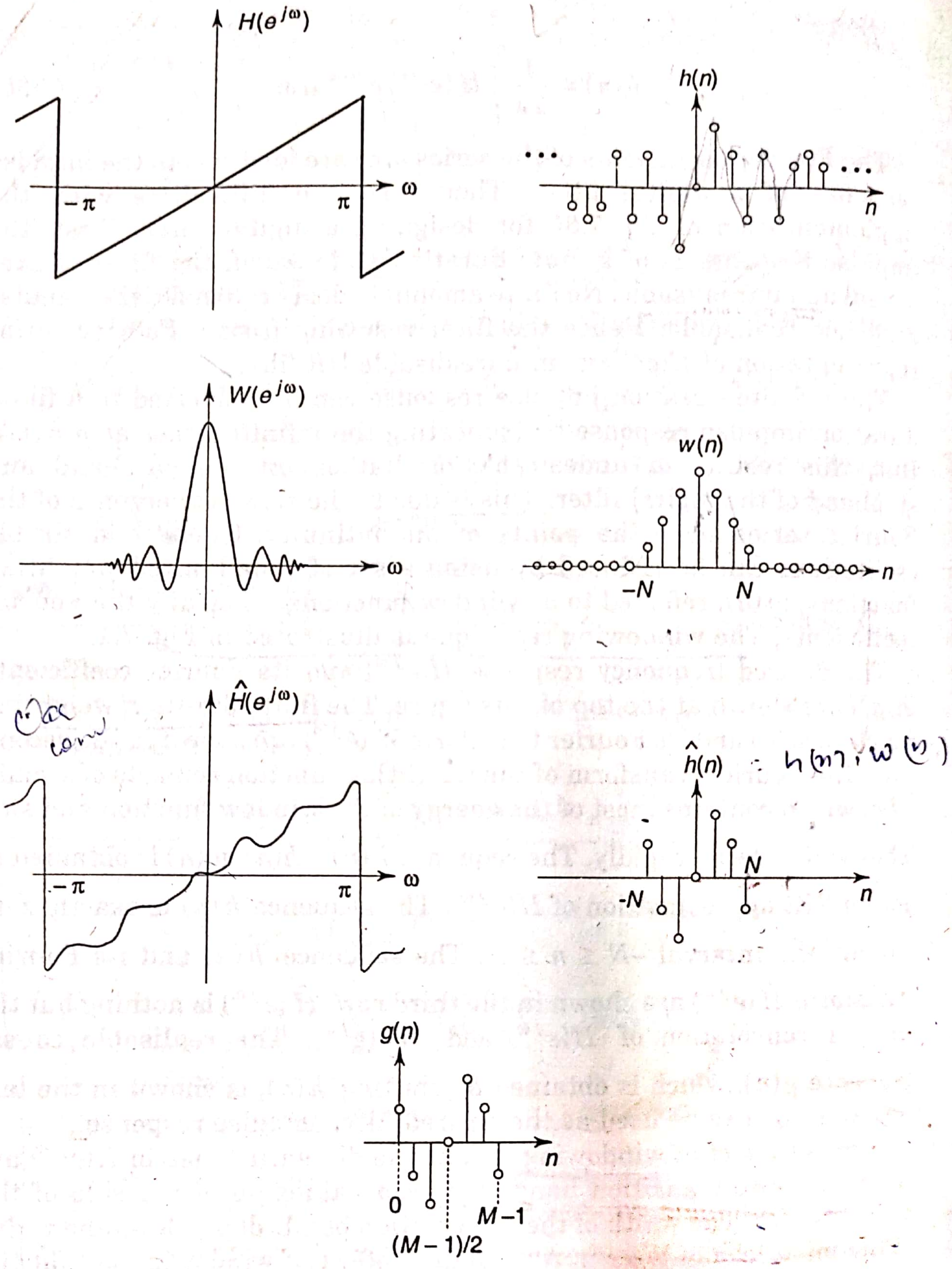

Fig. 7.1 *Illustration of the Window Technique*

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7.4.3.1 Rectangular Window Function

The weighting function for the rectangular window is given by

$$w_R(n) = \begin{cases} 1, & \text{for } |n| \le \dfrac{M-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

The spectrum of $w_R(n)$ can be obtained by taking Fourier transform of Eq. 7.37, as

$$W_R(e^{j\omega T}) = \sum_{n=\frac{-(M-1)}{2}}^{\frac{(M-1)}{2}} e^{-j\omega nT}$$

Substituting $n = m-(M-1)/2$ and replacing m by n, we get

$$W_R(e^{j\omega T}) = e^{j\omega\frac{(M-1)}{2}T} \sum_{n=0}^{(M-1)} e^{-j\omega nT}$$

$$= e^{j\omega\frac{(M-1)}{2}T} \cdot \frac{1-e^{-j\omega MT}}{1-e^{-j\omega T}} = \frac{e^{j\omega\frac{M}{2}T} - e^{-j\omega\frac{M}{2}T}}{e^{j\omega\frac{T}{2}} - e^{-j\omega\frac{T}{2}}}$$

Non Causal

$$= \frac{\sin\left(\frac{\omega MT}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)} \tag{7.38}$$

A sketch of Eq. 7.38 is shown in Fig. 7.2. The transition width of the main lobe is approximately $4\pi/M$. The first sidelobe will be 13dB down the peak of the main lobe and the rolloff will be at 20dB per decade. For a causal rectangular window, the frequency response will be,

$$W_R(e^{j\omega T}) = \sum_{n=0}^{(M-1)} e^{-j\omega nT}$$

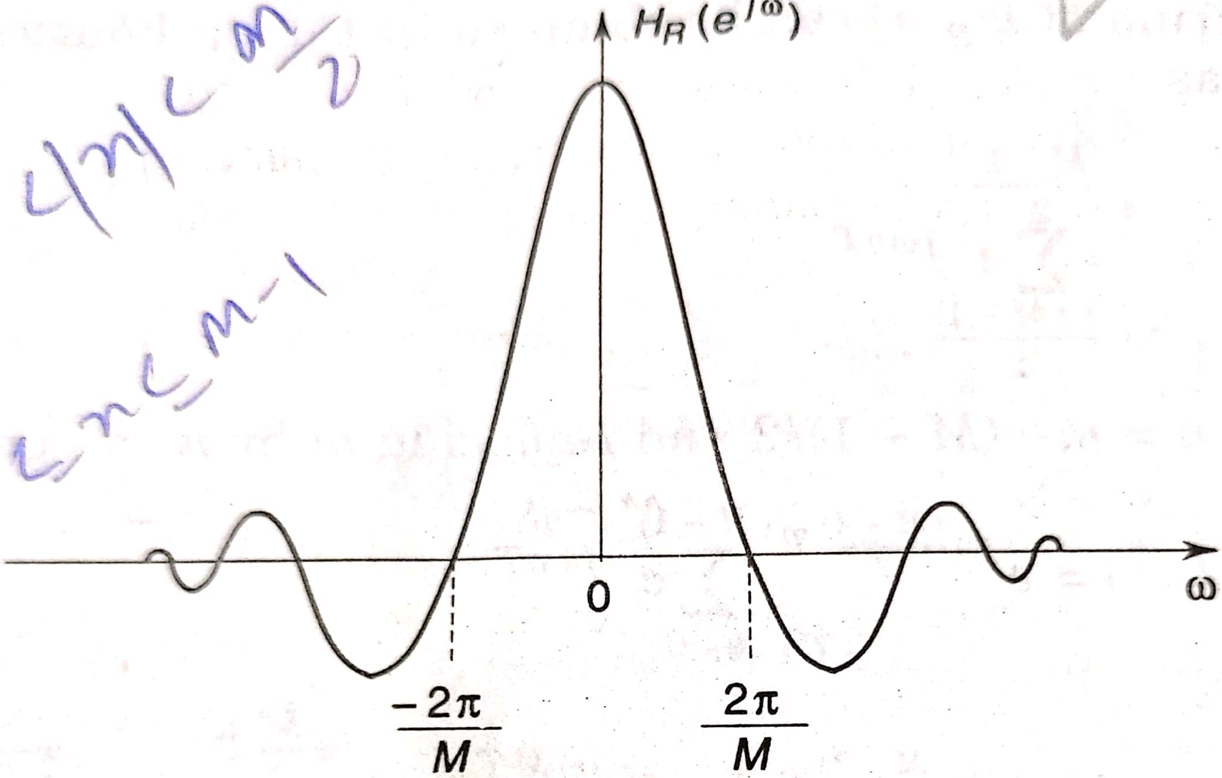
Causal

$$= e^{-j\omega\frac{(M-1)}{2}T} \frac{\sin\left(\frac{\omega MT}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)} \tag{7.39}$$

, $0 < |n| < \frac{M-1}{2}$

al

, $0 \le n \le M-1$



Hamming Window function

Causal Hamming window function is

$$W_H(n) = \begin{cases} 0.54 - 0.46 \cos \frac{2\pi n}{M-1}, & 0 \leq n < M-1 \\ 0, & \text{elsewhere} \end{cases}$$

Non-causal hamming window function is

$$W_H(n) = \begin{cases} 0.54 + 0.46 \cos \frac{2\pi n}{M-1}, & |n| \leq \frac{M-1}{2} \\ 0, & \text{elsewhere} \end{cases}$$

Non causal $W_H(n)$ & $W_R(n)$ (rectangular window function) are related as

$$W_H(n) = W_R(n) \left[0.54 + 0.46 \cos \frac{2\pi n}{M-1} \right]$$

$\therefore$ Spectrum of Hamming window will be

$$W_H(e^{j\omega T}) = 0.54 \frac{\sin\left(\frac{\omega M T}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)} + 0.46 \frac{\sin\left(\frac{\omega M T}{2} - \frac{M\pi}{M-1}\right)}{\sin\left(\frac{\omega T}{2} - \frac{\pi}{M-1}\right)} +$$

$$0.46 \frac{\sin\left(\frac{\omega MT}{2} + \frac{M\pi}{M-1}\right)}{\sin\left(\frac{\omega T}{2} + \frac{\pi}{M-1}\right)}$$

Main lobe width $\simeq \frac{8\pi}{M}$

Peak of first side lobe is at $-43\,dB$

Side lobe Roll off $= 20\,dB/decade$

Similarly, causal Hamming window spectrum will be

$$W_H(e^{j\omega T}) = 0.54 \frac{\sin\left(\frac{\omega MT}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)} - 0.46 \frac{\sin\left(\frac{\omega MT}{2} - \frac{M\pi}{M-1}\right)}{\sin\left(\frac{\omega T}{2} - \frac{\pi}{M-1}\right)}$$

$$- 0.46 \frac{\sin\left(\frac{\omega MT}{2} + \frac{M\pi}{M-1}\right)}{\sin\left(\frac{\omega T}{2} + \frac{\pi}{M-1}\right)}.$$

Hanning window function

Causal

$$w_{Hann}(n) = \begin{cases} 0.5 - 0.5\cos\dfrac{2\pi n}{M-1}, & 0 \le n \le M-1 \\ 0, & \text{elsewhere} \end{cases}$$

Non-causal

$$W_{Hann}(n) = \begin{cases} 0.5 + 0.5\cos\dfrac{2\pi n}{M-1}, & 0 < |n| < \dfrac{M-1}{2} \\ 0, & \text{elsewhere} \end{cases}$$

Main lobe width $= \dfrac{8\pi}{M}$

Peak of side lobe ~~is~~ at $-32\,dB$